Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name : Discrete Mathematics

Subject Code : 4TE04DSM2 **Branch: B. Tech (CE)**

Semester : 4 **Date : 15/04/2019** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Negation of $\exists x\ \forall y\ P(x,y)$ is
(A) $\forall x\ \forall y \sim P(x,y)$ (B) $\forall x\ \exists y \sim P(x,y)$ (C) $\exists x\ \exists y \sim P(x,y)$
(D) none of these

b) The negation of "some students like football" is

(A) Some students dislike football (B) Every student dislikes football

(C) Every student likes football (D) none of these

c) The number of binary relations of a set with n element is
(A) n^2 (B) 2^n (C) 2^{n^2} (D) none of these

d) The inverse of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$ is
(A) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}$ (B) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}$ (C) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}$
(D) none of these

e) In a group $(G, \circ)$ if $(a \circ b)^{-1} = a^{-1} \circ b^{-1}$ then
(A) G is finite (B) G is infinite (C) G is abelian (D) none of these

f) The maximum number of zero element and unit element each in a poset is
(A) 0 (B) 1 (C) 2 (D) none of these

g) A self-complemented, distributive lattice is called
(A) Boolean algebra (B) Modular lattice (C) Bounded lattice

(D) Complete lattice

h) In a lattice, if $a \le b$ and $c \le d$, then
(A) $b \le c$ (B) $a \le d$ (C) $a \vee c \le b \vee d$ (D) none of these

i) If B is a Boolean Algebra, then which of the following is true
(A) B is a finite but not complemented lattice.
(B) B is a finite, complemented and distributive lattice.
(C) B is a finite, distributive but not complemented lattice.
(D) B is not distributive lattice.

j) Let * be a Boolean operation defined by A * B = AB + $\bar{A}\,\bar{A}\;\bar{B}\,\bar{B}$, then A*A is:
(A) A (B) B (C) 0 (D) 1

k) A graph is a collection of
(A) Row and columns (B) Vertices and edges (C) Equations
(D) None of these

l) A tree is ___________.
(A) always disconnected graph (B) always a connected graph
(C) may be connected or disconnected (D) None of these

m) Pigeonhole principle states that $A \rightarrow B$ and $|A| > |B|$ then:
(A) f is not onto (B) f is not one-one (C) f is neither one-one nor onto
(D) f may be one-one

n) A debating team consists of 3 boys and 2 girls. Find the number of ways they can sit in a row?
(A) 120 (B) 24 (C) 720 (D) 12

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Show that $\sim r$ is a valid conclusion from the premises $p \Rightarrow \sim q,\ r \Rightarrow p,\ q$ (a) with truth table (b) without truth table. **(5)**

b) State and prove Lagrange's theorem on group. **(5)**

c) Draw Hasse diagram for the poset $\langle S_{24},\ \mathbf{D} \rangle$; where a**D**b means a divides b. **(4)**

Q-3 **Attempt all questions** **(14)**

a) Find all subgroup of cyclic group of order 12 with generator a. Also find the order of each element of group G and find other generators of G. **(5)**

b) Prove that $\langle \{1,\ 2,\ 3,\ 6\},\ \text{GCD, LCM} \rangle$ is a sublattice of the lattice $\langle S_{30},\ \text{GCD, LCM} \rangle$. **(5)**

c) Find Join-irreducible elements and atoms for the lattice $\langle S_4 \times S_9,\ \text{D} \rangle$. **(4)**

Q-4 **Attempt all questions** **(14)**

a) Using definition of complement of an element find complement of each element of lattice $\langle S_{10},\ \text{GCD, LCM, 1, 10} \rangle$ **(5)**

b)	Find all sub algebra of Boolean algebra $\langle S_{210},\ *, \oplus, ', 0,\ 1\rangle$.	**(5)**
c)	Draw all non-isomorphic graph on 2 and 3 vertices.	**(4)**

Q-5 **Attempt all questions** **(14)**

a) State and prove Stone's representation theorem. **(5)**

b) Draw the graph of tree represented by **(5)**

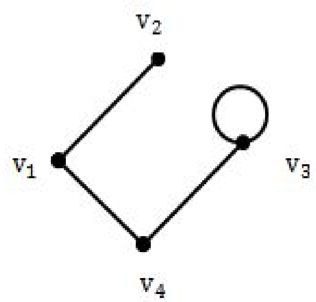
$$\left(v_0\ \left(v_1\ \left(v_2\right)\ \left(v_3\ \left(v_4\right)\ \left(v_5\right)\right)\right)\ \left(v_6\ \left(v_7\left(v_8\right)\right)\ \left(v_9\right)\ \left(v_{10}\right)\right)\right)$$

c) Show that $3+33+333+..........+33...........3=\left(10^{n+1}-9n-10\right)/27$ **(4)**

By mathematical induction.

Q-6 **Attempt all questions** **(14)**

a) Show that following graph is connected. **(5)**



b) Show that in any room of people who have been doing handshaking there will always be at least two people who have shaken hands the same number of times. **(5)**

c) Show that the following Boolean expression are equivalent. **(4)**

(i) $(x\oplus y)*(x'\oplus y),\ y$

(ii) $x*\left(y\oplus\left(y'*\left(y\oplus y'\right)\right)\right),\ x$

(iii) $(z'\oplus x)*\left((x*y)\oplus z\right)*(z'\oplus y),\ x*y$

Q-7 **Attempt all questions** **(14)**

a) Prove that "Is a lattice isomorphism" is an equivalence relation on the set of all lattices. **(5)**

b) From the following adjacency matrix, find the out degree and in degree of each node. Also verify your answer by drawing digraph and its adjacency matrix. **(5)**

$$\begin{array}{c c} & \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

c) Prove that the set of fourth roots of unity form a group under multiplication. **(4)**

Q-8 **Attempt all questions** **(14)**

a) Determine all the proper subgroup of symmetric group $(S_3, \cdot)$. Which subgroup is normal? **(5)**

b) Find all the minterms of a Boolean algebra with three variables x_1, x_2, x_3. **(5)**

c) Show that $(p \vee q) \wedge (\sim p \wedge \sim q)$ is a contradiction. **(4)**